Note presentations - 2 possible approaches

Math 5A Quiz 1.6 and 3.4 calculating limits

Show all work neatly with clear presentations. Name: ______________________

(2 points each part)

(1) Compute the following limits. Show work using algebraic methods, not by making a table of numbers or using a graph. After arriving at you answer, you are welcome to look at a graph for further exploration and understanding.

(a) $\lim_{x\to 4^-} \frac{x^2}{x-4}$ = $-\infty$

$\to \frac{16}{0}$ "$\frac{\text{nonzero}}{0}$"

$\Rightarrow \left\{ \begin{array}{l} \infty \\ -\infty \\ \text{dne} \end{array} \right.$ vertical asymptote

$x \to 4^- \Rightarrow x-4 < 0$

$\frac{+}{-}$

no equals here

(b) $\lim_{x\to 9} \frac{x-9}{3-\sqrt{x}}$ = -6

simplify - algebra first

$\frac{x-9}{3-\sqrt{x}} \cdot \frac{3+\sqrt{x}}{3+\sqrt{x}}$

$= \frac{-1\,(\cancel{x-9})(3+\sqrt{x})}{\cancel{9-x}}$

$= -(3+\sqrt{x})$

so

Then limit

$\lim_{x\to 9} \frac{x-9}{3-\sqrt{x}} = \lim_{x\to 9} -(3+\sqrt{x}) = -(3+\sqrt{9})$

$= -6$

write limit at each step until computed

(c) $\lim_{x\to 4} \frac{2x^2-7x-4}{x^2-16}$ = $\frac{9}{8}$

$= \lim_{x\to 4} \frac{(x-4)(2x+1)}{(x-4)(x+4)}$

$= \lim_{x\to 4} \frac{2x+1}{x+4}$

$= \frac{2(4)+1}{4+4} = \frac{9}{8}$

(d) $\lim_{h\to 0} \frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$ = $-2/x^3$

$\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h} = \frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h} \cdot \frac{(x+h)^2x^2}{(x+h)^2x^2}$

$= \frac{x^2-(x+h)^2}{h(x+h)^2x^2} = \frac{x^2-(x^2+2xh+h^2)}{h(x+h)^2x^2}$

$= \frac{-2xh+h^2}{h(x+h)^2x^2} = \frac{\cancel{h}(-2x+h)}{\cancel{h}(x+h)^2x^2}$

so

$\lim_{h\to 0} \frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h} = \lim_{h\to 0} \frac{-2x+h}{(x+h)^2x^2} = \frac{-2x}{x^2x^2} = \frac{-2}{x^3}$

(2) Given $f(x)=\begin{cases} x^2-3 & \text{if} \quad x>2 \\ x+1 & \text{if} \quad 0<x\le 2 \\ \dfrac{2x-1}{x-1} & \text{if} \quad x\le 0 \end{cases}$, find

$f(x)=\frac{2x-1}{x-1}$, $f(x)=x+1$, $f(x)=x^2-3$

0   2

$\lim\limits_{x\to 2} f(x)$ ___dne___ and $\lim\limits_{x\to 0} f(x)$ ___1___

If the limit DNE explain why.

$$\lim_{x\to 2^-} f(x)=\lim_{x\to 2^-}(x+1)=2+1=3$$

$$\lim_{x\to 2^+} f(x)=\lim_{x\to 2^+}(x^2-3)=4-3=1$$

Since $\lim\limits_{x\to 2^-} f(x)\neq \lim\limits_{x\to 2^+} f(x)$, $\lim\limits_{x\to 2} f(x)$ DNE

$$\left.\begin{aligned}\lim_{x\to 0^-} f(x)&=\lim_{x\to 0^-}\frac{2x-1}{x-1}=1\\ \lim_{x\to 0^+} f(x)&=\lim_{x\to 0^+}(x+1)=1\end{aligned}\right\}\text{equal}$$

(3) Compute the following limits. Show work.

Again, 2 possible ways to present

(a) $\lim\limits_{x\to\infty}\dfrac{3x^2}{x^3+2x+1}=$ ___0___

(b) $\lim\limits_{x\to\infty}\dfrac{\cos x}{x}=$ ___0___

$\lim\limits_{x\to\infty}\cos x$ DNE

" DNE "?? / ∞

$$\lim_{x\to\infty}\frac{3x^2}{x^3+2x+1}=\lim_{x\to\infty}\frac{3x^2}{x^3+2x+1}\cdot\frac{1/x^3}{1/x^3}$$

" ∞ " / ∞

$$=\lim_{x\to\infty}\frac{3/x}{1+2/x^2+1/x^3}=\frac{0}{1}=0$$

$$-1\le\cos x\le 1$$

$$-\frac{1}{x}\le\frac{\cos x}{x}\le\frac{1}{x}$$

squeeze thm

Since $\lim\limits_{x\to\infty}\left(-\frac{1}{x}\right)=\lim\limits_{x\to\infty}\left(\frac{1}{x}\right)=0$

$$\lim_{x\to\infty}\frac{\cos x}{x}=0$$

" ∞ " / ∞

(a) $\lim\limits_{x\to-\infty}\dfrac{2x^2-7x-4}{x^2-16}=$ ___2___

(b) $\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2+1}}{3x+6}=$ ___$-1/3$___

Show details so it is clear you understand

" ∞ " / ∞

$$\frac{2x^2-7x-4}{x^2-16}=\frac{2x^2-7x-4}{x^2-16}\cdot\frac{1/x^2}{1/x^2}$$

$$=\frac{2-7/x-4/x^2}{1-16/x^2}$$

So

$$\lim_{x\to-\infty}\frac{2x^2-7x-4}{x^2-16}=\lim_{x\to-\infty}\frac{2-7/x-4/x^2}{1-16/x^2}$$

$$=2$$

$$\lim_{x\to-\infty}\frac{\sqrt{x^2+1}}{3x+6}=$$

$$=\lim_{x\to-\infty}\frac{\sqrt{x^2(1+1/x^2)}}{3x+6}$$

$$=\lim_{x\to-\infty}\frac{|x|\sqrt{1+1/x^2}}{3x+6}=\lim_{x\to-\infty}\frac{(-x)\sqrt{1+1/x^2}}{3x+6}\quad\frac{1/x}{1/x}$$

Since $x\to-\infty$, $x<0$ so $|x|=-x$

$$=\lim_{x\to-\infty}\frac{-\sqrt{1+1/x^2}}{3+6/x}=-\frac{1}{3}$$